INNOVATION AND STOCK MARKET PERFORMANCE:

A MODEL WITH AMBIGUITY-AVERSE AGENTS

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Empirical evidence on stock prices shows that firms investing successfully in radical innovation experience higher stock returns. This paper provides a model that sheds light on the relationship between the degree of firm innovativeness and stock returns, whose movements capture expectations on firm's profitability and growth. The model grounds on the Neo-Schumpeterian growth models setup and relies on the crucial assumption of radical innovation characterized by "ambiguity" or Knightian uncertainty: due to its uniqueness and originality, no distribution of probability can be reasonably associated to radical innovation' success or failure. Extreme ambiguity (maxmin preferences) and smoother ambiguity aversion (α-maxmin, Choquet) are here compared. Results show that the assumption of ambiguity-aversion is critical in determining higher returns in presence of radical innovation and that the specific definition of expected utility shapes the extent of the returns.

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1. INTRODUCTION

The present paper focuses on the relationship between innovation and stock market returns, and investigates it through a model depicting innovation as an ambiguous decision. The intuition is the following: firms that are more R&D intensive are characterized by a higher degree of idiosyncratic risk: the more radical the innovative process, the stronger the uncertainty of expected future profits. Breakthrough innovation is characterized by Knightian uncertainty (Knight, 1921) or ambiguity, because no distribution of probability can be associated to the success of the investment in R&D. Investing in radical innovation has an uncertain outcome: it creates both favorable expectations for its future growth and fears that the investment will lead to a "dry hole".

Insofar, scholars have explained a firm's innovative attitude with its dimension and/or the intensity of market competition (Mazzucato, 2006). However, investing in innovation strongly affects the firm's stock value: the entrepreneur who wants to push the value of her firm upwards should enhance the firm's chances of future success, and being innovative is the main way to reach this goal. As asset pricing is a function of the stochastic discount factor which incorporates firm level risk, the revenues of highly innovative firms should be higher than non-innovative firms' revenues.

Managers and business strategists seems to be well-aware of this relationship. For instance, Forbes publishes every year a list of the most innovative companies by calculating the "innovation premium" as the proportion of a company's market value that cannot be accounted for from the net present value of cash flows of its current products in its current markets: it is the premium the stock market gives a company because investors expect it to launch new offerings and enter new markets that will generate even bigger income streams. Furthermore, management and strategy consulting firms like Booz & Company has consistently shown over the past years that there is no long-term correlation between the amount of money a company spends on its innovation efforts and its overall financial performance; instead, "what matters is how companies use that money and other resources, as well as the quality of their talent, processes, and decision making, i.e. how it deals with uncertain outcomes" (from *booz.com*).

Some previous works (Campbell and Shiller, 1988; Mazzucato and Semmler, 1999; Campbell, 2000; Mazzucato, 2003; Mazzucato and Tancioni, 2005; 2012) have emphasized the existence of a link between the degree of innovative disruptiveness and the volatility stock returns at the firm level. So far, very few theoretical studies have been devoted to the relationship between innovation and stock returns. I introduce a model that aims at summarizing the key mechanisms behind this link and that emphasizes the prominent role of ambiguity in affecting the decisions related to radical innovation and the consequences of agents' ambiguity aversion. The model compares ambiguity *à la* Gilboa-Schmeidler (1989) (maximin expected utility) to smoother forms of ambiguity-aversion as α-maxmin preferences introduced by Ghirardato et al. (2004) and Choquet expected utility in Schmeidler (1989)'s formulation. I also show that smoother ambiguity-aversion *à la* Klibanoff et al. (2005) in this specific case falls into the limiting case of maxmin preferences.

In the model, the introduction of a disruptive innovation is captured by allowing the firm to use a radically new input whose cost is sunk and whose returns are ambiguous: the aim is to disentangle the economic forces that determine the stochastic discount factor, that represents the reward that investors demand for bearing ambiguity.

The results show that the more a firm is innovative, the higher the idiosyncratic ambiguity level and the higher stochastic discount factor. Furthermore, the specific form of ambiguity strongly shapes the results.

2. RELATED LITERATURE

A radical innovation is a unique event that cannot be interpreted within a group of instances or in the light of similar occurrences (Knight, 1921): thus, it can be considered a good example of "Knightian uncertainty" or "ambiguity" that may end up into a market revolution but also into a dramatic and costly failure. Foreseeing the probability of success on the basis of R&D expenditure levels is a task that "can only partially be addressed by past data" (Athanassoglou et al., 2012). Uncertainty derives from several factors, such as the type of processed knowledge (e.g. Dewar and Dutton, 1986; Henderson, 1993), the interaction between firm-specific capabilities and institutions (e.g. Nord and Tucker, 1987) in determining the outcome, the difficulty to anticipate consumers' reaction and to figure out the eventual opening up of a new market and consequent applications (e.g. Henderson and Clark, 1990; O'Connor, 1998). Decision makers are typically much disturbed by ambiguous situations, and the empirical evidence shows pervasive ambiguity aversion (Ellsberg, 1961; Sarin and Weber,1993; Chesson and Viscousi, 2003; Gilboa, 2004), although more recent works question that its occurrence is systematic and relate it to specific contexts (e.g. Trautmann et al., 2008; Butler et al., 2011). If we account for ambiguity attitudes, investing in radical innovation is not only a consequence of evaluations on performance and costs, but might be dramatically affected by cognitive burdens. Managerial enquiries testify that ambiguity aversion, together with inertia and compartmentalized thinking, may constitute a learning barrier to the development of drastically new paths: firms tend to proceed as they always did, preserving the status quo rather than capitalizing on market information (Adams et al., 1998). Still, radical innovation occurs, and innovators with accumulated experience have been shown to be more efficient in searching and combining knowledge components (Fleming, 2001).

Interest in radical innovations is due not only to firms perspective profits and market share, but also to their importance in determining the dynamic of expected long-run growth. Stock prices reflect these expectations, that are in general based on fundamentals, but also affected by irrational exuberance, bandwagon phenomena, herd behaviors, and over-reactions. Mazzucato (2006) reviews the main results on the empirical relationship between innovation and the volatility of stock returns, and observes that “there is a missing link between the industrial economics literature on innovation and uncertainty and the finance literature on risk and the volatility of stock prices. There are, however, various studies that focus on the effect of innovation on the level of stock prices. Jovanovic and MacDonald (1994) relate the evolution of the average industry stock price level to the current stage of the industry life-cycle: they claim that the average stock price falls just before the shakeout occurs because a disruptive innovation causes a sudden drop in present product price which is detrimental for incumbents. Jovanovic and Greenwood (1999) link stock prices to innovation in a model where innovation causes new capital to destroy old capital: since it is incumbents who are quoted on the stock market, innovations by new firms determines an immediate decline in the stock market because investors with perfect foresight anticipate this damage to old capital. Proxying innovative input with patents, Pakes (1985) shows that unexpected changes in patents and in R&D are associated with relevant changes in the market value of the firm, although in presence of large variance that may reflect an extremely dispersed distribution of the values of patented ideas.

In general, the empirical evidence shows a relationship between stock prices and successful innovation having a positive impact on a firm’s profits and growth, consistently with the idea that stock prices reflect expectations about discounted future profits. Furthermore, in phases characterized by radical innovation, firms that are seen as both probable winners and losers will experience volatility in their stock prices (Pastor and Veronesi, 2006). Uncertainty about a firm’s average future profitability, that can be thought as uncertainty about the average future growth rate of a firm’s book value, increases a firm’s fundamental value (Pastor and Veronesi, 2003). This is because innovation often causes a shake-up of market shares, diminishing the power of the incumbents who have an incentive to preserve the status quo. In this situation, current performance is not a good indicator of future performance: investors are more likely to be influenced by the speculation of other investors, leading to high volatility (Campbell and Shiller, 1981).

3. THE MODEL

This model is grounded on Romer's (1994) and Aizenman's (1997) neo-Schumpeterian models of growth in their closed-economy version, enriched by assumptions on agents' ambiguity-aversion. I consider three different specifications of preferences in case of ambiguity that reflect Gilboa and Schmeidler (1989), Ghirardato et al. (2004), and Choquet (1957) - in Schmeidler (1989)'s formulation - expected utility definitions.

Neo-Schumpeterian models explicitly allow for an introduction into an economy of new or improved types of goods: their peculiarity consists of taking explicit account of the fixed costs that limit the set of goods and of showing that these fixed costs matter in a dynamic analysis conducted at the level of the economy as a whole. This contrasts with the standard approach in general equilibrium analysis, in which fixed costs are assumed to be of negligible importance in markets. These models of endogenous growth theory differ from the models in Romer (1986, 1987, 1990) and Lucas (1988), which emphasize external increasing returns, and from the models in Jones and Manuelli (1990) and Rebelo (1991), which are grounded on perfect competition and assume that capital can be accumulated forever without driving its marginal product to zero; both the external effects and perfect competition models of endogenous growth assume that new goods do not matter at the aggregate level Furthermore, new growth models also depart from the literature in industrial organization because they do not capture explicitly the strategic interactions that emerge when there are only a small number of firms in a market.

The crucial premise in neo-Schumpeterian models is that every economy faces virtually unlimited possibilities for the introduction of new goods, where the term "good" is used in the broadest possible sense: it might represent an entirely new type of physical good, or a quality improvement; it might be used as a consumption good, or as an input in production. Here, the introduction of a new capital good represents an innovation.

The firm goes through two periods: in period *0*, it decides whether innovate or not, and (if it is the case) sustains the sunk costs needed for a breakthrough innovation; in period *1*, production takes place. We consider an innovating firm which produces a final good *Z* by using labour *L* and *N* capital goods x_i according to the following production function:

$$Z = (L)^{1-\beta} \sum_{i=1}^{N} (x_i)^{\beta} \qquad (1)$$

with $0 \leq \beta \leq 1$. The production of capital good x_n takes place using the services of labour according to the function $x_n = L_n$ where L_n stands for the labour in activity n, whereas L is the labour employed in production of the final good. For simplicity, as standard in this literature, w is the real wage and represents the marginal cost of producing both the capital goods and the final good.

The new capital good n can be introduced either as a small improvement in the existing technology (incremental innovation) or as a disruptive opening up of a new technology (radical innovation).

Standard cost minimization implies that the demand for capital good i is:

$$(x_i)^d = \left(\frac{\beta}{p_i}\right)^{1-\beta} L \tag{2}$$

Each producer faces a demand whose elasticity is $\frac{1}{1-\beta}$.

3.1 INNOVATION WITH SUBJECTIVE EXPECTED UTILITY

The most familiar model of choice under uncertainty follows Savage (1954) in assuming that agents maximize expected utility according to subjective priors (subjective expected utility, henceforth SEU): agents are uncertain about payoffs, but there is no uncertainty about the model and the probabilities associated to each state of the world is known. This means that agents are not equipped to distinguish between risk (known probabilities) and ambiguity (unknown probabilities): agents who maximize SEU exhibit no care about ambiguity.

If the innovator were a SEU agent, she would assign a uniform distribution to the returns of innovation. The only information available is that the project return is bounded between L and H, where L < H. The expression $\frac{L+H}{2}$ represents the expected return of the investment in innovation, where the probability assigned to the unsuccessful outcome L is $\frac{1}{2}$: an ambiguity-neutral Bayesian agent will refer to this expression as the expected return. For sake of simplicity, I assume that agents assign to both success and failure the same probability[1].

A representative producer of the x_i capital good follows a mark-up rule, charging $p_i = \frac{w}{\beta}$ for its input. Adding capital good n will lead to profits equal to

[1] Assuming different probabilities does not affect the results.

$$\pi_n(\chi) = \frac{\chi(\mathrm{p_i}-\mathrm{w})\mathrm{x_i}}{1+r} - n = \frac{\chi\,(\mathrm{w})^{-\frac{\beta}{1-\beta}}}{1+r}\frac{1-\beta}{\beta}\beta^{\frac{2}{1-\beta}}L - n = \frac{\chi\,\mathrm{W}}{1+r} - n \qquad (3)$$

where $k = \frac{1-\beta}{\beta}\beta^{\frac{2}{1-\beta}}$, $\beta' = \frac{\beta}{1-\beta}$ and $W = (\mathrm{w})^{-\beta'}\mathrm{kL}$

Investment in a radical innovation will be undertaken if it increases the expected utility deriving from the introduction of the new capital good:

$$E_u(\pi\,) = \frac{W}{1+r} - n > 0 \qquad (4)$$

that leads to $n < \frac{W}{1+r}$.

The interpretation is the following: a new capital good is introduced only if its cost is lower than the expected profit, that must be discounted at a rate *r*. Since SEU agents do not care about the ambiguous returns of radical innovation, the same condition holds both when the introduction of capital good *n* is an improvement of the existing technology and when it opens a new technological trajectory. If so, improving the existent technology and introducing in a totally new one makes no difference from the decision-maker point of view: from an investor's point of view stock returns have the same stochastic discount factor (henceforth SDF), no matter how conservative or disruptive is a firm's strategy. This result is summarized in the proposition below.

Proposition 1. *With SEU preferences, the investor evaluates projects by applying an ambiguity-free SDF denoted by* r *and invests in innovation when the following condition holds:* $n < \frac{W}{1+r}$.

As the empirical evidence strongly departs from this result, I assume preferences that accounts for ambiguity-aversion. The next paragraph presents one form of extreme ambiguity-aversion (maxmin preferences *à la* Gilboa-Schmeidler).

3.2 INNOVATION WITH MAX-MIN AMBIGUITY

In this sections, I depart from SEU assuming that the investors bear the ambiguity of the returns of each state and are characterized by a strong form of ambiguity aversion on the set of logically possible priors. To capture agents' attitude towards risk and uncertainty, I first follow the Gilboa-

Schmeidler or maximin approach, that assumes an extreme type of ambiguity-aversion (Gilboa and Schmeidler, 1989). As emphasized by Ellsberg (1961), agents are unable to summarize uncertainty in the form of a unique prior distribution. Therefore, they attach an extra cost to invest in a radical innovation that might be interpreted as an "ambiguity premium". In the Schmeidler-Gilboa approach, ambiguity induces ambiguity-averse innovators to prefer more transparent information and therefore to discount by using a "hurdle rate" that is higher than the ambiguity-free interest rate. The introduction of a new capital good *n* by means of a radical innovation requires an "up-front capacity investment" which is specific to the new capital good, whereas the marginal cost of all the current capital goods is equal to w. Adding capital good *n* requires a sunk cost specific to that good; the innovator commits to its investment at the beginning of period *0*, whereas production takes place in period *1*. For simplicity, I assume that the dependence of the sunk cost on *n* is linear and is normalized at *1* (I assume it is known). On the contrary, future revenues are uncertain due to the fact that the new technological trajectory may or may not be successful (and this is not known *a priori).* I label χ the random shock that describes the degree of uncertainty of the innovation that affects future revenues and has mean μ equal to 1. For simplicity, I normalize χ to be either low ($\chi = 1 - \delta$) or high ($\chi = 1 + \delta$), $\delta \geq 0$. Subjects assume the probability assigned to both the successful and unsuccessful outcomes (H and L respectively) is $\frac{1}{2}$ and is independent of the degree of ambiguity about the innovation outcomes. The precise probability of each state is unknown and δ represents the range of possible outcomes of the random variable χ, i.e. ambiguity.

Improvements in the existing technology are assumed to involve no ambiguity in the profitability of the technology: in this case, $\delta = 0$ and $\chi = 1$. Therefore, in case of incremental innovation, the investor evaluates projects by applying the ambiguity-free interest rate r .

Investing in disruptive innovation exposes the innovator to ambiguity. When assuming maxmin expected utility (henceforth MEU), the decision rule can be depicted as to maximize a utility index that provides a proper weight for the exposure to ambiguity: agents maximize expected utility according to the belief which generates the lowest utility. The procedure I follow consists of constructing two statistics. The first is the "worst scenario" wealth, denoted by $\underline{\pi}$. The second is the "expected wealth" if one attaches a uniform prior to the distribution of the profits, denoted by $E_u(\pi)$. The shortcoming of $E_u(\pi)$ is that it does not give any weight to the ambiguity regarding the outcome of the innovation. To correct this shortcoming, I use a decision rule that maximizes the innovator's utility U as a weighted average of the above two statistics:

$$U = \alpha\underline{\pi} + (1-\alpha)E_u(\pi) \qquad (5)$$

where α represents the degree of maxmin ambiguity-aversion embodied in the decision to invest, with $0 \leq \alpha \leq 1$. When α goes to zero, we have the case of a ambiguity-neutral Bayesian agent who attributes a uniform prior to the two events. A larger α indicates higher ambiguity-aversion.
Investment in a disruptive innovation will be undertaken if it increases the expected utility:

$$\alpha \frac{(1-\delta)W}{1+r} + (1-\alpha)\frac{W}{1+r} - n > 0 \qquad (6)$$

that leads to $n < \frac{W}{1+\bar{r}}$, where $(1+\bar{r}) = \frac{1+r}{1-\alpha\delta} > 1+r$.

In case of extreme ambiguity aversion (e.g. when α=1) , the expression above becomes:

$$\frac{(1-\delta)W}{1+r} - n > 0$$

It means that the decision maker accounts only for the worst scenario.
On the contrary, in case of ambiguity neutrality (e.g. when α=0), the expression above becomes

$$\frac{W}{1+r} - n > 0$$

i.e. the agent takes her decision on the basis of subjective expected utility (SEU).

Proposition 2. *With MEU preferences, an innovative firm's SDF is higher than a conservative firm's SDF and is equal to* $(1+\bar{r}) = \frac{1+r}{1-\alpha\delta} > 1+r$.

The SDF is a random variable whose realization is always positive: it generalizes the notion of discount factor to an uncertain world. If there is no ambiguity (as in case of incremental innovation), or if investors are ambiguity-neutral, the SDF is just a constant *r* that converts future expected payoffs into present value. The assumption of SEU agents would not allow to capture a difference in returns between disruptive and innovative firms.

The immediate consequence of Proposition 2 is that stocks of a firm investing in radical innovation promise higher returns than stocks of a more conservative firm: this occurs in order to pay back investors of their capability to bear ambiguity with respect of investments that do not involve ambiguous outcomes (see Corollary I). This result is consistent with the empirical evidence and strongly relays on the assumption of agents' maxmin ambiguity aversion (see Corollary II).

Corollary I to Proposition 2. *In case of incremental innovation, a maxmin ambiguity-averse agent's SDF is* $\bar{r} = r$.

Proof. As an investment in incremental innovation does not imply ambiguity, we get this inequality by assuming $\delta = 0$ (due to $\chi = 1$). ■

Corollary II to Proposition 2. *In case of radical innovation, a maxmin ambiguity-neutral agent's SDF is* $\bar{r} = r$.

Proof. As an investment in incremental innovation does not imply ambiguity, we get this inequality by assuming $\alpha = 0$. ■

The following proposition focuses on the relationship between ambiguity, ambiguity-aversion and level of stock returns.

Proposition 3. *With MEU preferences, the SDF rises in ambiguity and in ambiguity aversion.*

Proof. It is straightforward to see that the more innovation is disruptive and "vague" in its returns, the higher the SDF; the higher the degree of ambiguity aversion, the higher the SDF. ■

This assumption of maxmin preferences deserves further comments. With the specification introduced in the model, I define δ as the degree of vagueness, i.e. the amount of "objective" ambiguity on the possible values the random shock assumes. An additional interpretation can be in terms of volatility of stock returns (see the discussion below). On the other hand, the parameter α captures the "subjective" attitude towards ambiguity. Although the assumption on MEU is inspired by Gilboa and Schmeidler (1989)'s preferences, this model is able to distinguish between ambiguity and ambiguity-aversion.

A typical critique to Gilboa and Schmeidler (1989)'s model is that it implies extreme ambiguity aversion, or even "paranoia" (Epstein and Schmeidler, 2010). Klibanoff et al. (2005;

2009) present a model with smoother ambiguity where agents' preferences are built such that the agent computes the certainty equivalent over all the possible state of nature and takes the minimum. The utility function can be solved in two stages: first, the expected utilities are calculated for all the priors in the corresponding set and a set of expected utilities is obtained. Second, the distorted expectation described above is taken by aggregating a transformation of these expected utilities with respect to the second order prior, i.e., the updated belief over the latent state. The transformation of the expected utilities captures the agent's ambiguity attitude; in particular, if the transformation is concave then the agent is ambiguity averse while if it is affine then the agent is ambiguity neutral and simply maximizes a subjective expected utility. Since I assume utility to be a linear function of profits, ambiguity *à la* Klibanoff et al. (2005) falls into maxmin expected utility. The following section describes an alternative form of smoother ambiguity attitude.

3.3 INNOVATION WITH α-MAXMIN EXPECTED UTILITY

In this section, I generalize the previous findings by following Ghirardato et al. (2004)'s specification. It broadens the spectrum of agents' behaviour traits with a small departure from MEU preferences and with little loss in terms of tractability. The decision maker do not account for the worst scenario only, but also for the best one; however, ambiguity-aversion α stands out from agent's overweight of the worst outcome.

The agent maximizes the following statistics:

$$U = \alpha\underline{\pi} + (1-\alpha)\overline{\pi} \tag{7}$$

where α here represents the degree of α–maxmin ambiguity aversion embodied in the decision to invest, with $0 \leq \alpha \leq 1$. When α equals 1, we have the case of extreme ambiguity-aversion of maxmin expected utility preferences. When α equals 1/2, we have the case of ambiguity-neutrality as in subjective expected utility preferences. When α equals 0, we have the case of extreme ambiguity-seeking behaviour. In contrast to the previous, this formulation also accounts for ambiguity-seeking preferences.

Investment in a disruptive innovation will be undertaken if it increases the expected utility:

$$\alpha\left[\frac{(1-\delta)W}{1+r}\right] + (1-\alpha)\left[\frac{(1+\delta)W}{1+r}\right] - n > 0 \tag{8}$$

that leads to $n < \frac{W}{1+\tilde{r}}$, where $(1+\tilde{r}) = \frac{1+r}{1+\delta(1-2\alpha)} > 1+r$.

In case of extreme ambiguity-aversion (e.g. when α =1) , the expression above becomes:

$$\pi_0 + \frac{(1-\delta)W}{1+r} - n > \pi_0$$

that leads to the same results that MEU preferences.

In case of ambiguity-neutrality (e.g. when α=1/2), the expression above becomes

$$\pi_0 + \frac{W}{1+r} - n > \pi_0$$

i.e. the agent takes her decision on the basis of subjective expected utility (SEU).

In case of ambiguity-seeking (e.g. when α=0), the expression above becomes

$$\pi_0 + \frac{(1+\delta)W}{1+r} - n > \pi_0$$

i.e. the agent accounts for the best scenario and has a discount factor $\tilde{r}$ such that $(1+\tilde{r}) = \frac{1+r}{1+\delta} < 1+r$, i.e. exhibit a SDF than is lower than the ambiguity-free SDF.

Proposition 4. *An agent with α-MEU preferences exhibits a smoother form of ambiguity-aversion than MEU preferences and, ceteris paribus, faces a lower SDF $\tilde{r}$ such that* $(1+\tilde{r}) = \frac{1+r}{1+\delta(1-2\alpha)} > 1+r$.

The α-maxmin model encompasses both the case of MEU preferences when α=1 (extreme ambiguity aversion), and SEU preferences when δ=0 (no ambiguity).

3.4 INNOVATION WITH CHOQUET EXPECTED UTILITY

The following paragraph presents an alternative specification where ambiguity implies non-additive probabilities. Schmeidler (1989)'s version of Choquet expected utility model differs from Savage's expected utility model in not necessarily assuming probability to be additive: agent's beliefs to be represented by a unique but non-additive probability. Schmeidler referred to them as non-additive probabilities, and required that they be positive and monotone with respect to set inclusion. Such mathematical entities are also known by the term "capacities": the capacity in the model can be interpreted as a lower bound on probabilities.

The agent maximizes the following statistics:

$$U = \frac{1}{2}\underline{\pi} + \left(\frac{1}{2} - \alpha\right)\overline{\pi} \tag{9}$$

where α is the degree of pessimism in the assumed probability distribution. Consequently, *(1-α)* represents the degree of confidence. Note that $\mathbf{0} \leq \boldsymbol{\alpha} \leq \frac{\mathbf{1}}{\mathbf{2}}$.

When α equals 0, we have the case of ambiguity-neutrality as in subjective expected utility preferences; when α equals ½, we have the case of extreme ambiguity-aversion of maxmin expected utility preferences. In general, a larger α indicates less confidence about the assigned probabilities and greater ambiguity aversion.

Investment in a radical innovation will be undertaken if it increases the expected utility:

$$\frac{1}{2}\left[\frac{(1-\delta)(w)^{-\beta}kL}{1+r}\right] + \left(\frac{1}{2} - \alpha\right) \left[\frac{(1+\delta)(w)^{-\beta}kL}{1+r}\right] - n \quad > 0 \tag{10}$$

that leads to $n < \frac{W}{1+\hat{r}}$ where $(1+\hat{r}) = \frac{1+r}{1-\alpha(1+\delta)}$.

Having either ambiguity or ambiguity aversion equal to zero leads to SEU preferences. It is the interaction between ambiguity and ambiguity-aversion that determines the SDF, but underconfidence in the model matters *per se*.

When α=1 (extreme pessimism), the SDF is negative: an investors should be paid to invest, we have the extreme case of no trading. In general, a positive discount factor and consequently trade are possible if $\delta < \frac{1}{\alpha} - 1$, i.e. when the degree of ambiguity is not too large with respect to the amount of ambiguity that agents are able to tolerate.

In this framework, the impact of perceived ambiguity on the expected returns from innovation expresses the nature and intensity of the psychological bias revealed by decision makers under ambiguity, that might be called α–ignorance.

Proposition 4. *With Choquet ambiguity-averse agents, an innovative firm's SDF is higher than a conservative firm's SDF.*

Proof. It is straightforward to see that the SDF increases in the degree of ambiguity δ; the higher the degree of ambiguity-aversion α, the higher the effect of δ on the SDF. ■

Interestingly, in case of Choquet preferences, the SDF is higher than SDF with SEU preferences also in presence of incremental innovation, i.e. unambiguous investment.

Proposition 5. *With Choquet ambiguity-averse agents, the lower the degree of confidence, the higher the SDF.*

Proof. The SDF increases in the degree of pessimism in the correctness of the model $(1-\alpha)$; in case of extreme optimism, the SDF turns to the SDF we get in case of SEU preferences. ■

The last proposition emphasizes two key findings: first, innovators who appear very confident in their knowledge of a new technology do not apply a hurdle rate when discounting objectively ambiguous profits. Second, when investors feel extremely optimistic in their knowledge of the model, there is no need of compensating agents for an ambiguous investment and there could be no difference in stock revenues of high innovative and conservative firms.

Some previous versions of Choquet expected utility involve distorting probability measure: if the distortion function is concave, then the least favourable events receive increased weight and the most favourable events are discounted reflecting pessimism. Thus, instead of the uniform weighting implicit in the expected utility criterion and in this version of Choquet preferences, other models accentuate the weight of the least favourable events and reduce the weight assigned to the most favourable events or, alternatively, exaggerate the likelihood of the more favourable events and downplays the likelihood of the worst outcome.

4.COMPARISON AMONG AMBIGUITY MODELS

The sections above presented three models of decision making that allow for non-neutral approaches to ambiguity. The question is now how should we select the model to work with when investigating the relationship between stock returns and firm innovativeness. As emphasized by Gilboa and Marinacci (2011), there are alternative approaches to this problem. First, one may compare the different models by a "horse-race": the model that best explains the observed phenomenon should be used for prediction. Alternatively, in the light of the theoretical difficulties in selecting a specific model, one may try to obtain general conclusions within a class of models, without committing to a particular theory of decision making. This approach has been suggested in the context of risk by Machina (1982). In this well-known paper, Machina has shown that, for some applications, economists need not worry about how people really make decisions, since a wide range of models were compatible with particular qualitative conclusions. A similar approach has been suggested for decisions under uncertainty. An example of this approach is the notion of biseparable preferences, as in Ghirardato and Marinacci (2002): biseparable preferences assume smoothness and monotonicity and include both Choquet preferences and maxmin preferences. Ghirardato and Marinacci (2001) provide a definition of ambiguity aversion that does not depend on the specific model of decision making and applies to all biseparable preferences. This allow for a general approach to preferences under ambiguity which, similarly to Machina (1982), remains silent regarding the actual structure of preferences, thereby offering a highly flexible model. In this perspective, α-maxmin preferences appear to be general enough to encompass both the case of ambiguity neutrality and maximum ambiguity aversion and allow for an interpretation in terms of confidence in the decision model. Furthermore, they are compatible also with ambiguity seeking attitude: as shown in Section 2, the empirical evidence presents several situation and contexts where decision makers seek for ambiguity. In case of disruptive firms, positive announcements on perspective profits, like news on financial results and sales might be interpreted as signals for success in radical innovation and cause investors to reduce ambiguity aversion and increase the demand for stocks.

5. DISCUSSION AND CONCLUSIONS

There is overall agreement that radical innovation is important (e.g. Leifer et al., 2001): consensus has emerged that conventional incremental improvements and cost reduction strategies are insufficient for obtaining a competitive advantage (Sorescu et al., 2003) as a direct consequence of worldwide diffusion of knowledge and industrial capability. Understanding radical innovation may eventually make their course shorter, less sporadic, less expensive. Furthermore, understanding radical innovation may shed light on stock market prices both in terms of level and volatility. The empirical evidence shows that firms' innovativeness is positively related to firms' value as measured by stock returns. This is due to the compensation that investors need to get when bearing the ambiguity involved in radical innovation. Ambiguity characterizes radical innovations as opposed to incremental innovations, where only measurable uncertainty is involved.

The paper presents a neo-Schumpeterian model that accounts for the introduction of new goods and captures the related sunk costs. The crucial hypotheses we introduce are that (a) radical innovation is an ambiguous decision, and (b) investors are ambiguity-averse. We suggest three possible ways of capturing ambiguity. The first, based on Gilboa and Schmeidler (1989)'s approach - implies an extreme form of ambiguity aversion; the second, based on Ghirardato et al. (2004)'s approach, presents smoother ambiguity and allows for ambiguity seeking behaviour too; the third, based on Schmeidler (1989)'s version of Choquet's notion of capacity, interprets ambiguity aversion as underconfidence on the correctness of the model the decision makers use to interpret the real world.

The results show that it is ambiguity aversion that makes the difference between radical and incremental innovation so crucial: if agents were ambiguity-neutral, then radical innovation would not bring about higher stock prices than incremental innovation. In presence of ambiguity, modelled as a larger set of possible priors, firms will be more willing to invest in incremental innovation rather than bet on investing on more disruptive ones, and investors should be compensated for their investment in stocks whose returns are ambiguous. In a Subjective Expected Utility model, the firm's probability of being successful in introducing a radical innovation would be known, and the investor would switch, at a certain price, from demanding a this firm's stocks to offering them. This is no longer the case when the probability of success is not known. In this case, in presence of ambiguity averse investors, there will be an interval of prices at which neither buying nor selling will seem attractive, and an ambiguity averse agent will choose to hold an unambiguous portfolio of stocks of a more conservative firm. In particular, an agent who is maximally ambiguity will always choose to hold an unambiguous portfolio, no matter the relative prices of stocks. By contrast, an agent who maximizes expected utility with respect to a subjective prior will choose to hold equal quantities of two stocks only if the ratio of prices is equal to the ratio of subjective probabilities.

This may explain both why people refrain from trading in certain markets, and why entrepreneurs exhibit inertial behaviour with respect of engaging in the exploration of new technologies. It can also explain why at times of higher volatility one may observe lower volumes of trade: with a larger set of probabilities that are considered possible, there will be more investors who decide neither to buy nor to sell.

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